Analysis of Gerrymandering in Ohio Using Metric Geometry

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Mathematics

Gerrymandering

- Gerrymandering is the practice of drawing electoral districts to intentionally favor a certain political party
- Can be done to show partisan bias or racial bias
- Only three federally enforced regulations:
  - Districts must be similar in population size
  - Districts must be contiguous
  - Districts must be “reasonably compact”

Figure 1: The original gerrymander printed in a March 1812 newspaper. This district was drawn to favor Governor Elbridge Gerry

Methods of Gerrymandering

- Majority-Minority Districts-
  - Made to increase minority representation within Congress
- Packing and Cracking

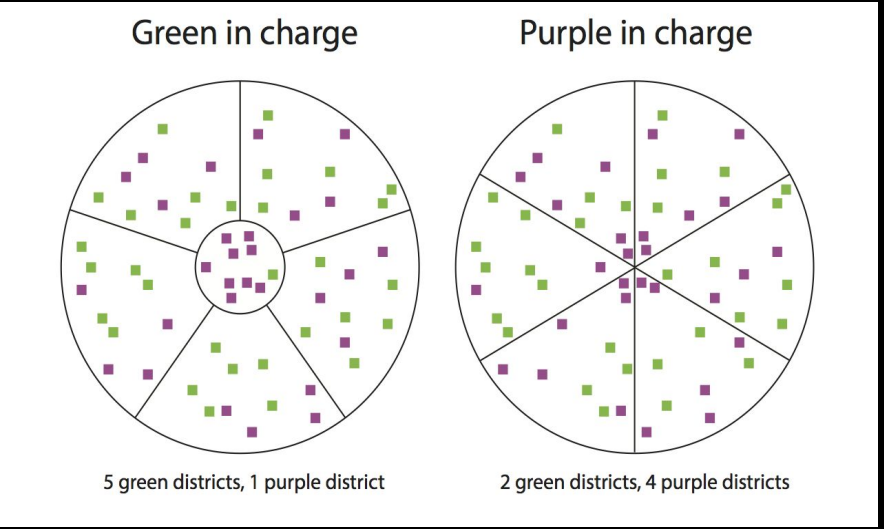

Figure 2: Image depicting packing and cracking

Benisek v. Lamone

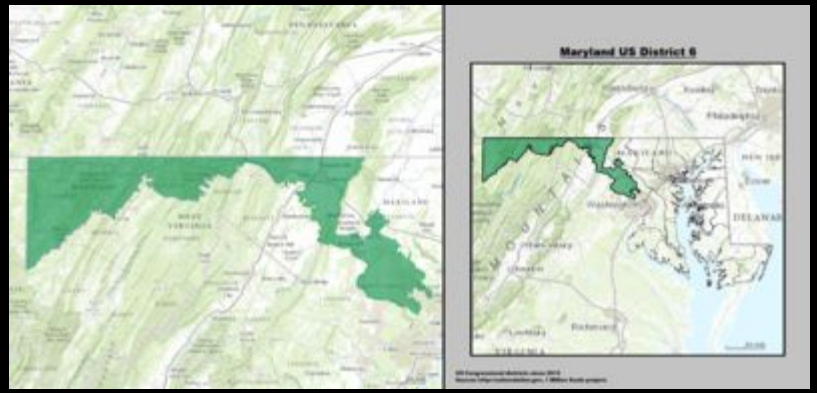

Figure 3: Maryland District 6 that was brought to court

- Plaintiffs filed complaint that Maryland had been partisan-gerrymandered on November 5, 2013
- In statewide elections, Democrats won 7 out of 8 seats available while winning only 63% of the vote
- Presented to Supreme Court on September 1, 2017
- Court granted the plaintiff's request that the congressional map be redrawn before the 2020 election

Ohio

- Consistently accused of being heavily gerrymandered
- Swing state in federal elections; it is often said that whichever candidate wins Ohio in the electoral college, wins the presidency
- Republicans could win the 75% of the House seats with only 50% of the vote
- Republicans could win a House majority with only 42% of the vote

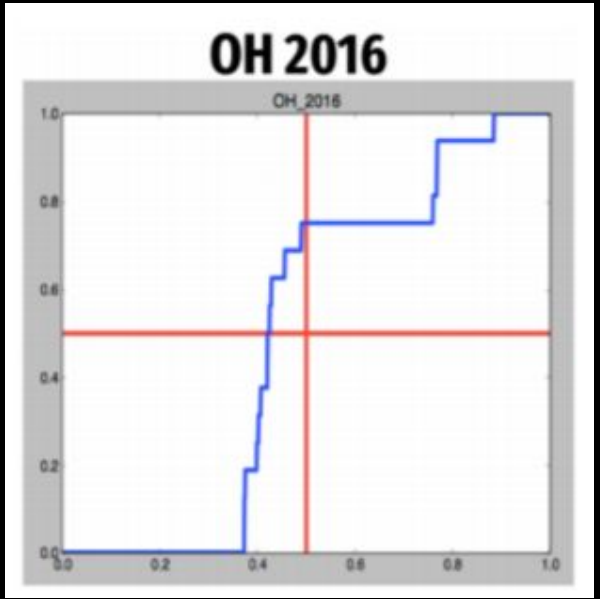

Figure 4: Image capturing the lack of symmetry of votes to representation within Ohio.

Geometry

- The clause of districts needing to be, “reasonably compact,” is often overlooked due to its loose definition.
- Gerrymandering is usually analyzed in the statistical sense
- Rarely analyzed geometrically
- Ohio district-drawing process:
  - House and Senate majority-leaders each choose two legislators of the General Assembly and one non-legislature to be a part of the committee that draws the districts
  - Once a plan is drawn, it is sent to the General Assembly and governor for approval.

Figure 5: District plan for Ohio 2012-2022

Reock Test

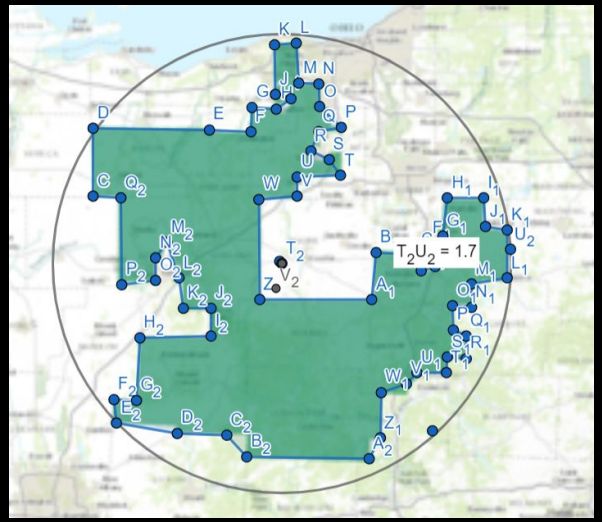

Figure 7: Ohio Congressional District 7 with the minimum bounding circle in the Geogebra app.

Ratio of area of district to area of minimum circumscribing circle:

$$0 \leq \frac{Area\ of\ District}{Area\ of\ Minimum\ Bounding\ Circle} \leq 1$$

Polsby-Popper Test

Compares the area of the district to the area of a circle with the same perimeter:

$$0 \leq \frac{4\pi \; x \; Area \; of \; District}{(Perimeter)^2} \leq 1$$

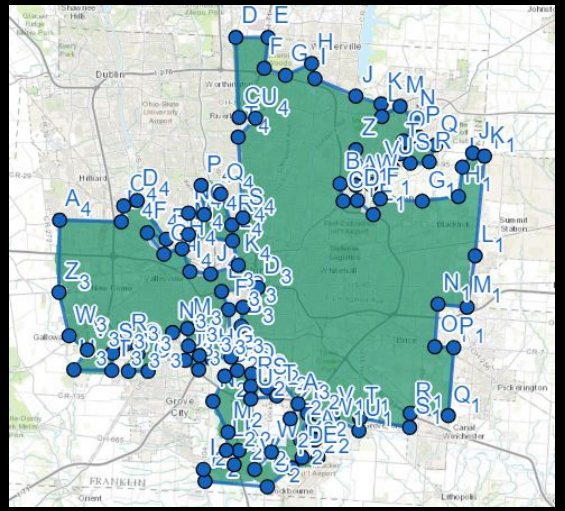

Figure 8: US Ohio Congressional District 3 outlined in the Geogebra app.

Convex Hull Test

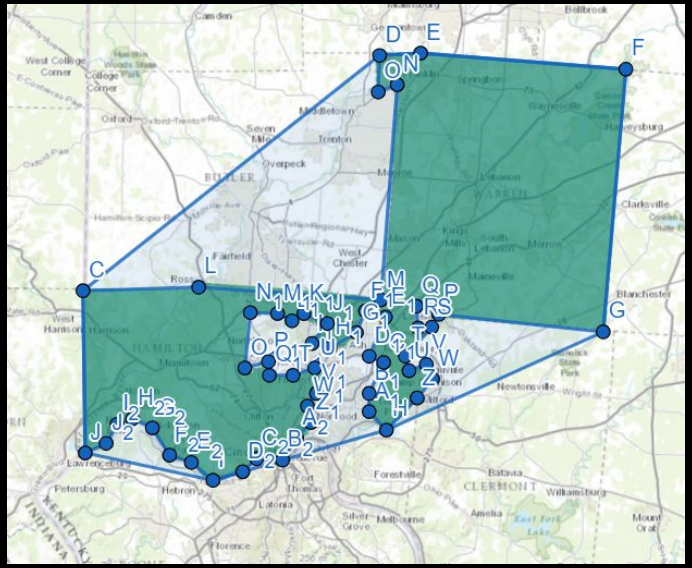

Figure 9: The convex hull compactness test applied to US Congressional District 1 in the Geogebra app.

Ratio of the area of district to area of the minimum bounding convex polygon:

$$0 \leq \frac{Area\ of\ District}{Area\ of\ Minimum\ Bounding\ Convex\ Shape} \leq 1$$

Self-Devised Test

Comparison of the geometric centers of the district itself and the minimum circumscribing circle.

$$0 \leq \frac{Radius - Distance}{Radius} \leq 1$$

In this equation, the *Radius* is the radius of the minimum circumscribing circle and the *Distance* is the distance between the geometric centers of both the district and circle

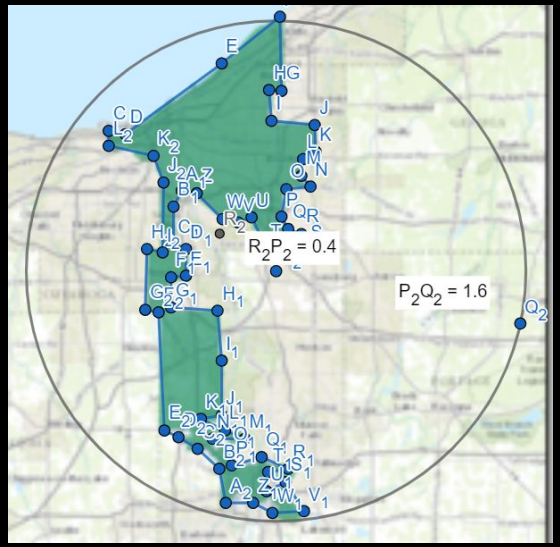

Figure 10: US Congressional District 11 outlined in the Geogebra app.

Data

	Minimum	Maximum	Median	Mean
Reock	0.2181	0.6090	0.3281	0.3536
Polsby-Popper	0.0621	0.4435	0.2040	0.2301
Convex Hull	0.5000	0.8571	0.6309	0.6593
Self-Devised	0.8000	1.000	0.9070	0.8950

Table 1: The minimum and maximum scores along with the median and mean of test.

Results

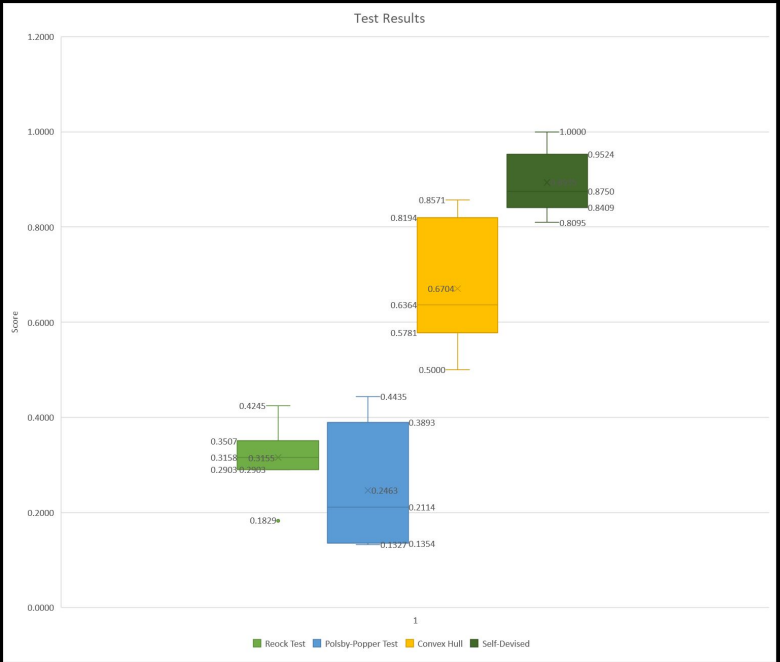

Figure 11: Box and whisker plots detailing the ranges of the test results

	Reock	Polsby-Popper	Convex Hull	Self-Devised
District 1				
District 2				
District 3				
District 4				
District 5				
District 6				
District 7				
District 8				
District 9				
District 10				
District 11				
District 12				
District 13				
District 14				
District 15				
District 16				

Figure 12: Chart indicating if a district passed each test. Black denotes that the district did not pass the test

Conclusions

- Geometry is important to consider when evaluating a district plan for gerrymandering
- Based on the inconsistency of these test results, districts cannot be analyzed with geometry alone
- In future research, other methods of measuring compactness could be used

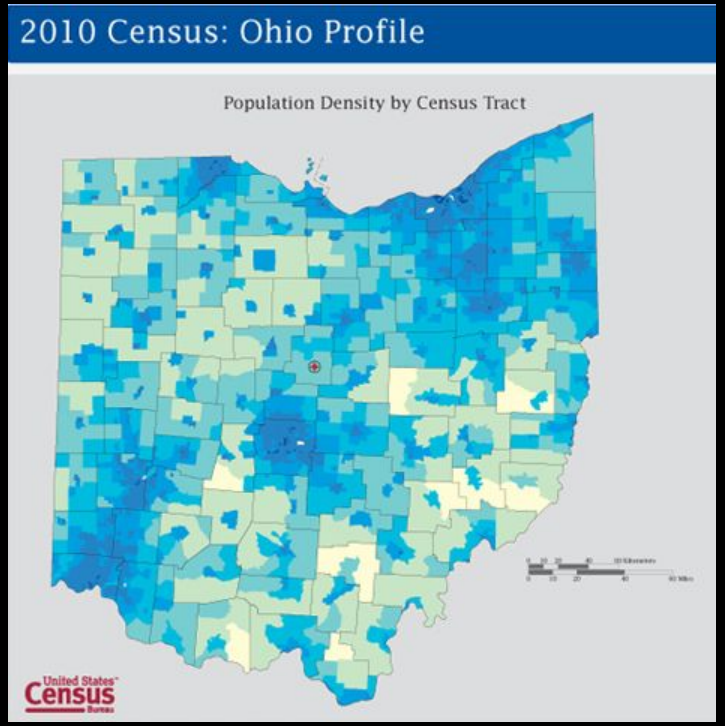

Figure 13: Image of the population density of Ohio by census tract.

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